

After the previous focus group rejected their last product, the company which manufactures plant-based foods changed the recipe for their synthetic meat product and conducted another focus group. The members of this focus group were each given a 4 ounce portion of the synthetic meat. Members were then randomly selected from the group, and  $X$  is the random variable representing the amount of the meat product the member ate (measured in ounces). The probability density function for  $X$  is

$$f(x) = \begin{cases} k \sin \frac{\pi x}{8}, & x \in [0, 4] \\ 0, & x \notin [0, 4] \end{cases} \text{ (for some appropriate constant } k \text{).}$$

- [a] Find the probability that a randomly chosen member of the focus group ate at least half the portion of the synthetic meat.  
**Your final answer must be a number, not an integral.**

$$\textcircled{4} \int_0^4 k \sin \frac{\pi x}{8} dx = 1$$

$$k \int_0^4 \sin \frac{\pi x}{8} dx = 1$$

$$\textcircled{5} k \cdot \left. \frac{-8}{\pi} \cos \frac{\pi x}{8} \right|_0^4 = 1$$

$$k \cdot \frac{-8}{\pi} (\cos \frac{\pi}{2} - \cos 0) = 1$$

$$k \cdot \frac{-8}{\pi} (0 - 1) = 1$$

$$\textcircled{2} k = \frac{\pi}{8}$$

$$P(2 \leq X \leq 4) = \int_2^4 \frac{\pi}{8} \sin \frac{\pi x}{8} dx \textcircled{4}$$

$$= \left. \frac{\pi}{8} \cdot \frac{-8}{\pi} \cos \frac{\pi x}{8} \right|_2^4 \textcircled{2}$$

$$= -(\cos \frac{\pi}{2} - \cos \frac{\pi}{4})$$

$$= -(0 - \frac{\sqrt{2}}{2})$$

$$= \frac{\sqrt{2}}{2} \textcircled{3}$$

- [b] Find the median amount of meat product eaten by a member of the focus group.  
**Your final answer must be a number, not an integral.**

$$\int_0^{X_{\text{MED}}} \frac{\pi}{8} \sin \frac{\pi x}{8} dx = \int_{X_{\text{MED}}}^4 \frac{\pi}{8} \sin \frac{\pi x}{8} dx = \frac{1}{2} \textcircled{4}$$

$$\textcircled{2} \left( -\cos \frac{\pi x}{8} \right) \Big|_{X_{\text{MED}}}^4 = \frac{1}{2}$$

$$-(\cos \frac{\pi}{2} - \cos \frac{\pi X_{\text{MED}}}{8}) = \frac{1}{2}$$

$$\textcircled{3} \cos \frac{\pi X_{\text{MED}}}{8} = \frac{1}{2}$$

$$\frac{\pi X_{\text{MED}}}{8} = \frac{\pi}{3} \textcircled{2}$$

$$X_{\text{MED}} = \frac{8}{3} \text{ OUNCES}$$

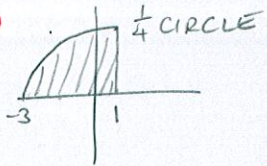
$$\textcircled{2} \quad \textcircled{2}$$

Find the value of  $c$  guaranteed by the Mean Value Theorem for Integrals for  $f(x) = \sqrt{16 - (x-1)^2}$  on  $[-3, 1]$ . SCORE: \_\_\_\_ / 20 PTS

$$f_{\text{AVE}} = \frac{1}{1-(-3)} \int_{-3}^1 \sqrt{16 - (x-1)^2} dx \quad (6)$$

$$= \frac{1}{4} \cdot \frac{1}{4} \pi 4^2 \quad (3)$$

$$= \pi \quad (2)$$



$$\sqrt{16 - (c-1)^2} = \pi \quad (4)$$

$$16 - (c-1)^2 = \pi^2$$

$$(c-1)^2 = 16 - \pi^2$$

$$c-1 = \pm \sqrt{16 - \pi^2}$$

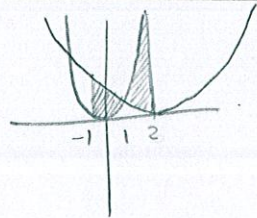
$$c = 1 \pm \sqrt{16 - \pi^2} \quad (3)$$

$$c = 1 - \sqrt{16 - \pi^2} \in (-3, 1) \quad (2)$$

Find the area between the curves  $y = 4x^2$  and  $y = (x-3)^2$  over the interval  $[-1, 2]$ .

SCORE: \_\_\_\_ / 25 PTS

Your final answer must be a number, not an integral. HINT: The answer is NOT 9.



$$4x^2 = (x-3)^2$$

$$4x^2 = x^2 - 6x + 9$$

$$3x^2 + 6x - 9 = 0$$

$$3(x+3)(x-1) = 0$$

$$x = -3, 1$$

$$\begin{aligned} & \textcircled{3} \int_{-1}^1 ((x-3)^2 - 4x^2) dx + \textcircled{3} \int_1^2 (4x^2 - (x-3)^2) dx \\ &= \int_{-1}^1 (-3x^2 - 6x + 9) dx + \int_1^2 (3x^2 + 6x - 9) dx \\ &= (-x^3 - 3x^2 + 9x) \Big|_{-1}^1 + (x^3 + 3x^2 - 9x) \Big|_1^2 \\ &= \textcircled{4} -1 - 3 + 9 - (-1 - 3 - 9) + (8 + 12 - 18) - (1 + 3 - 9) \\ &= 5 - -11 + 2 - -5 \\ &= \textcircled{3} \underline{23} \end{aligned}$$

Find the area of the surface created by revolving the arc of  $f(x) = \sqrt{x} - \frac{x^{\frac{3}{2}}}{3}$  on  $[1, 3]$  about the  $x$ -axis.

SCORE: \_\_\_\_ / 30 PTS

Your final answer must be a number, not an integral.

$r=y$

$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{2}x^{\frac{1}{2}} \quad (4)$$

$$(3) \quad 2\pi \int_1^3 \left(x^{\frac{1}{2}} - \frac{1}{3}x^{\frac{3}{2}}\right) \sqrt{1 + \left(\frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{2}x^{\frac{1}{2}}\right)^2} dx \quad (4)$$

$$= 2\pi \int_1^3 \left(x^{\frac{1}{2}} - \frac{1}{3}x^{\frac{3}{2}}\right) \sqrt{1 + \frac{1}{4}x^{-1} - \frac{1}{2} + \frac{1}{4}x} dx$$

$$= 2\pi \int_1^3 \left(x^{\frac{1}{2}} - \frac{1}{3}x^{\frac{3}{2}}\right) \sqrt{\frac{1}{4}x^{-1} + \frac{1}{2} + \frac{1}{4}x} dx$$

$$= 2\pi \int_1^3 \left(x^{\frac{1}{2}} - \frac{1}{3}x^{\frac{3}{2}}\right) \left(\frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}x^{\frac{1}{2}}\right) dx \quad (4)$$

$$= 2\pi \int_1^3 \left(\frac{1}{2} + \frac{1}{2}x - \frac{1}{6}x - \frac{1}{6}x^2\right) dx$$

$$= 2\pi \int_1^3 \left(\frac{1}{2} + \frac{1}{3}x - \frac{1}{6}x^2\right) dx \quad (4)$$

$$= 2\pi \left(\frac{1}{2}x + \frac{1}{6}x^2 - \frac{1}{18}x^3\right) \Big|_1^3 \quad (4)$$

$$= \pi \left(x + \frac{1}{3}x^2 - \frac{1}{9}x^3\right) \Big|_1^3$$

$$= \pi \left(3 + 3 - 3 - \left(1 + \frac{1}{3} - \frac{1}{9}\right)\right)$$

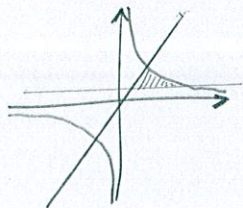
$$= \frac{16\pi}{9} \quad (3)$$

The region bounded by  $y = 2x$ ,  $y = \frac{8}{x}$  and  $y = 1$  is revolved about the line  $y = 8$ .

SCORE: \_\_\_\_\_ / 40 PTS

Find the volume of the resulting solid.

**Your final answer must be a number, not an integral.**



$$2x = \frac{8}{x}$$

$$x^2 = 4$$

$$x = \pm 2$$

$$y = \pm 4$$

$$\textcircled{3} \quad 2\pi \int_1^4 (8-y) \left( \frac{8}{y} - \frac{1}{2}y \right) dy$$

$$= 2\pi \int_1^4 \left( \frac{64}{y} - 4y - 8 + \frac{1}{2}y^2 \right) dy$$

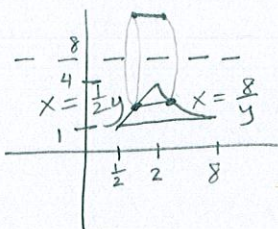
$$= 2\pi \left( 64 \ln|y| - 2y^2 - 8y + \frac{1}{6}y^3 \right) \Big|_1^4$$

$$= 2\pi (64(\ln|4| - \ln|1|) - 2(16-1) - 8(4-1) + \frac{1}{6}(64-1))$$

$$= 2\pi (64 \ln 4 - 30 - 24 + \frac{21}{2})$$

$$= \pi (128 \ln 4 - 87)$$

$\textcircled{4}$



OR

ALTERNATE (NOT RECOMMENDED)  
SOLUTION

$$\begin{aligned}
& \pi \int_{\frac{1}{2}}^2 \left( (8-1)^2 - (8-2x)^2 \right) dx + \pi \int_2^8 \left( (8-1)^2 - \left(8 - \frac{8}{x}\right)^2 \right) dx \\
& \stackrel{\textcircled{1\frac{1}{2}} \textcircled{3}}{=} \pi \int_{\frac{1}{2}}^2 (49 - (64 - 32x + 4x^2)) dx + \pi \int_2^8 (49 - (64 - \frac{128}{x} + \frac{64}{x^2})) dx \\
& \stackrel{\textcircled{7\frac{1}{2}}}{=} \pi \int_{\frac{1}{2}}^2 (-15 + 32x - 4x^2) dx + \pi \int_2^8 (-15 + \frac{128}{x} - \frac{64}{x^2}) dx \\
& \stackrel{\textcircled{3}}{=} \pi \left( -15x + 16x^2 - \frac{4}{3}x^3 \right) \Big|_{\frac{1}{2}}^2 + \pi \left( -15x + 128 \ln|x| + \frac{64}{x} \right) \Big|_2^8 \\
& = \pi \left( -30 + 64 - \frac{32}{3} - \left( -\frac{15}{2} + 4 - \frac{1}{6} \right) - 120 + 128 \ln 8 + 8 - \left( -30 + 128 \ln 2 + 32 \right) \right) \\
& = \pi (128 \ln 4 - 87) \textcircled{4}
\end{aligned}$$